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THEORETICAL CONSIDERATIONS REGARDING THE BEHAVIOUR OF A MINE MODEL WITH VENTILATION UNDER TIME-DEPENDENT RADIATION CONDITIONS

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THEORETICAL CONSIDERATIONS REGARDING THE BEHAVIOUR OF A MINE MODEL
WITH VENTILATION UNDER TIME-DEPENDENT RADIATION CONDITIONS

by

J. Bigu*

ABSTRACT

A mine model with ventilation for uranium mines has been investigated in which the rate of emanation of radon and thoron gas through mine walls, by diffusion, varies with time in a predictable fashion. Sinusoidal, linear and exponential cases are dealt with and expressions are derived to calculate the atom concentration of radioactive gas and its progeny. The expressions given can be used to determine the activity and Working Level, and other radiation variables and/or ratios of practical interest. In general terms, it is shown that the radiation variables indicated above follow a similar, although somewhat modified, dynamic behaviour as that corresponding to the time-dependent rate of emanation of radioactive gas. As expected, the solutions given here are proportional to the 'unperturbed' emanation rate.

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OBSERVATIONS THÉORIQUES SUR LE COMPORTEMENT D'UN MODÈLE DE MINE AVEC
VENTILATION DANS DES CONDITIONS DE RAYONNEMENT EN FONCTION DE TEMPS

par

J. Bigu*

RÉSUMÉ

Ce rapport présente une étude d'un modèle de mine avec ventilation pour mines d'uranium, dans lequel le taux d'émanation par diffusion des gaz de radon et de thoron à travers les parois de la mine varie avec le temps de manière prévisible. On a examiné les cas de diffusion sinusoidale, linéaire et exponentielle desquels on a tiré des formules servant à calculer la concentration atomique des gaz radioactifs et de leurs produits de filiation. Les formules obtenues peuvent être utilisées pour déterminer l'activité et l'unité alpha (Working Level) ainsi que les autres variables ou rapports de rayonnement d'intérêt pratique. On a de plus démontré d'une manière générale, que les variables de rayonnement indiquées ci-dessus suivent un comportement dynamique semblable, bien qu'un peu modifié, correspondant au taux d'émanation des gaz radioactifs en fonction de temps. Tel qu'attendu, les solutions présentées ici sont proportionnelles au taux d'émanation "non perturbé".

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INTRODUCTION

The growth of radon decay products in a radon atmosphere such as that of, say, an underground uranium mine has been considered in detail by several authors (1 to 7). Studies on this topic have resulted in the proposal of several mine models. An extension of these models to include thoron, a common occurrence in Ontario uranium mines, has been investigated by Bigu (7). A basic, simplifying, assumption made in the study of the dynamic behaviour of mine models is that the ventilation rate (air flow) and/or radon and thoron emanation from walls, if applicable to the particular mine model, remains constant. However, emanation from walls is known to be inversely related to barometric pressure, and ventilation changes are quite a common occurrence. Because in actual practice both variables can vary appreciably with time, it is of great practical interest to determine the behaviour of mine models under time-dependent conditions. Although in some cases underground pressure and ventilation characteristics are changed by the operator in a predictable fashion for some specific purpose, in other cases these changes simply occur without planning and/or operator intervention. It is, therefore, difficult under the latter conditions to investigate the behaviour of mine models. The behaviour of time-dependent mine models is best studied by assuming predictable time-dependent variations in the radon and thoron emanation rate from mine walls and/or of mine air flow.

This paper presents analytical expressions which describe the behaviour of a mine model in which the radon and thoron emanation rate from mine walls, and the mine air flow varies with time in the following predictable fashion: sinusoidally, linearly, exponentially and stepwise. Some quantitative theoretical data showing the dynamic behaviour of the mine model under a variety of time-dependent situations have been presented elsewhere (8).

THEORETICAL CONSIDERATIONS

The mine model under consideration here is the mine-tunnel model with ventilation (MTMV) initially proposed by Beckman and Holub (6) for radon, and later extended by Bigu (7) to the case of thoron, and to arbitrary mixtures of radon and thoron. In this paper, the MTMV is further extended to include time-dependent perturbations in the rate of emanation of radon and/or thoron from mine walls, and in the air flow, i.e., ventilation rate, through the mine.

The atom concentration N , i.e., number of atoms of a given atomic species per unit volume, of the radioactive parent gases radon, Rn-222, and thoron, Rn-220, and their daughter products, can be calculated by solving the following set of differential equations:

$$dN_n/dt = k_n(t) - \Lambda_n(t)N_n \quad \text{Eq 1}$$

for the radioactive gas, indicated by the subindex n , and

$$dN_i/dt = \lambda_{i-1}N_{i-1} - \Lambda_i(t)N_i \quad \text{Eq 2}$$

for the decay product of i -type.

In Equations 1 and 2 the symbols $\Lambda_n(t)$ and $\Lambda_i(t)$ are given by:

$$\Lambda_n(t) = \lambda_n + \lambda_v(t) \text{ and } \Lambda_i(t) = \lambda_i + \lambda_v(t)$$

where λ_n and λ_i are the radioactive half-lives of the parent gas and i -type daughter species, respectively. The symbol $\lambda_v(t)$ is given by $\lambda_v(t) = Q(t)/V$ where $Q(t)$ is the time-dependent air flow through a given volume of the mine, and V is the volume of this section of the mine. (It should be noted that the index $i-1$ indicates the parent of i , and that λ_v^{-1} is the mine air residency time.) The symbol $k_n(t)$ in Equation 1 stands for the time-dependent rate of emanation of radon and/or thoron through the mine walls into the volume of the mine under consideration. The effect of both $k_n(t)$ and $Q(t)$ has been investigated for the MTMV.

It should be noted that, for the sake of generality k_n and Λ in

Equations 1 and 2 are shown as time-dependent. In this paper, however, the effect of only $k_n(t)$ or $\Lambda(t)$, but not both simultaneously, has been investigated. Hence, the reader should be aware that when $k_n(t)$ is considered, Λ , i.e., λ_v , is assumed constant, and vice versa. The combined, simultaneous, effect of $k_n(t)$ and $\Lambda(t)$ is beyond the scope of this paper.

The following time-dependent perturbation cases, and some combinations of these perturbations, have been investigated:

$$\left. \begin{aligned} F(t) &= F_0(1 + \gamma \sin \omega t), \\ F(t) &= F_0(1 + \eta t), \\ F(t) &= F_0 e^{\delta t}, \text{ and} \\ F(t) &= q F_0. \end{aligned} \right\} \quad \text{Eq 3}$$

where, $q = 1$ for $t \leq t_0$, and

$$q \neq 0, 1 \text{ for } t > t_0$$

In the above equations $F(t)$ can apply to $k_n(t)$ and $Q(t)$. F_0 is some 'steady-state', i.e., constant, value for these variables before the time-dependent perturbation is in effect.

The equations above can be written in a more compact form. Thus:

$$F(t) = F_0 K(t) \quad \text{Eq 4}$$

The function $K(t)$ can be thought of as a perturbation introduced to the otherwise undisturbed, or steady-state, condition. The symbols γ , η and δ are the perturbation amplitudes. The symbol ω represents angular frequency.

It should be noted that for the initial boundary condition at $t = 0$, Equations 3 reduce to F_0 . The same holds true for $\gamma = 0$, $\eta = 0$ and $\delta = 0$.

The perturbation amplitudes can take any arbitrary positive or negative numerical value. For small values of δ , the second and third of Equations 3 are identical for $\eta = \delta$, as:

$$d^{\eta \delta} \approx 1 + \eta t \text{ for } \eta t \ll 1$$

Large values for δ are of little practical interest as they rarely reflect actual field conditions. Hence, the exponential perturbation can often be used instead of the linear perturbation with a considerable simplification of the analytical procedure.

Once N_n and N_i have been calculated from Equations 1 and 2, respectively, taking into account the applicable boundary conditions, the activities A_n and A_i can easily be determined by means of $A_n = N_n \lambda_n$ and $A_i = N_i \lambda_i$. The daughter activity permits, in turn, the calculation of the daughter Working Level, WL.

Analytical solutions for the relevant radioactive variables N_n , N_i , A_n , A_i , WL_i , and other variables, are given below for the case where $F(t) = k_n(t)$. The analytical solutions for the case where $F(t) = Q(t)$ are, however, extremely difficult to obtain. In the latter case, the solutions to Equations 1 and 2 have been obtained by numerical methods using a special Runge-Kutta procedure and the modified predictor-corrector method for the solution of linear initial-value problems. The numerical solutions for these cases are presented elsewhere (9).

In some instances, e.g., the sinusoidal and linear cases, a considerable simplification of the analytical procedure can be attained by calculating the contributions to N_n and N_i in Equations 1 and 2, respectively, from the unperturbed and time-dependent terms separately, and adding up the two contributions afterwards. This simplifying procedure has a two-fold advantage. First, it reduces the length of the calculation drastically by dividing it into two parts, and secondly, there is no need to calculate the contribution from the unperturbed case as this has already been dealt with elsewhere (7). The details of this simplifying approach will be given in a forthcoming report. Furthermore, the effect of more than one kind of perturbation, e.g., sinusoidal and exponential, acting simultaneously can easily be investigated by adding the separate contributions from the two perturbations together.

In its simplest form the step-function case can be represented by the transient associated with the unperturbed case, i.e., $N_n = N_i = 0$ at $t = 0$ at which time Equations 1 and 2 are solved for $k_n > 0$. However, a more representative and general case is that where k_n varies in a stepwise fashion after the transient referred to above reaches the steady-state condition, or, alternatively, during the transient. As before, the principle of linear superposition can be applied to the expressions derived for the unperturbed case (7).

In the following sections the index i will be substituted by A for RaA and ThA , B for RaB and ThB , and C for RaC and ThC .

SINUSOIDAL PERTURBATION

The solutions for $k_n(t) = k_0(1 + \gamma \sin \omega t)$ are given below.

$$N_n = N_n^0 + \frac{k_0 \gamma}{\Lambda_n^2 + \omega^2} (\Lambda_n \sin \omega t - \omega \cos \omega t + \omega e^{-\Lambda_n t}) \quad \text{Eq 5}$$

where N_n^0 represents the solution to the unperturbed case for N_n given by (7)

$$N_n^0 = (N_n)_0 e^{-\Lambda_n t} + \frac{k_0}{\Lambda_n} (1 - e^{-\Lambda_n t}) \quad \text{Eq 6}$$

where $(N_n)_0$ is the initial gas atom concentration at $t = 0$.

$$N_A = N_A^0 + \frac{k_0 \gamma \lambda_n}{\Lambda_n^2 + \omega^2} \left[\frac{\Lambda_n}{\Lambda_A^2 + \omega^2} (\Lambda_A \sin \omega t - \omega \cos \omega t) - \frac{\omega}{\Lambda_A^2 + \omega^2} (\Lambda_A \cos \omega t + \omega \sin \omega t) + \frac{\omega}{\Lambda_A - \Lambda_n} e^{-\Lambda_n t} \right] + D e^{-\Lambda_A t} \quad \text{Eq 7}$$

$$\text{where, } D = \frac{k_0 \gamma \lambda_n}{\Lambda_n^2 + \omega^2} \left[\frac{\omega (\Lambda_n + \Lambda_A)}{\Lambda_A^2 + \omega^2} - \frac{\omega}{\Lambda_A - \Lambda_n} \right] \quad \text{Eq 8}$$

and where N_A^0 represents the solution for N_A for the unperturbed case given by (7):

$$N_A^0 = \frac{k_0 \lambda_n}{\Lambda_n} \left[\frac{1}{\Lambda_A} (1 - e^{-\Lambda_A t}) - \frac{1}{\Lambda_A - \Lambda_n} (e^{-\Lambda_n t} - e^{-\Lambda_A t}) \right] + A \quad \text{Eq 9}$$

$$\text{where, } A = \frac{(N_n)_0 \lambda_n}{\Lambda_A - \Lambda_n} (e^{-\Lambda_n t} - e^{-\Lambda_A t}) + (N_A)_0 e^{-\Lambda_A t} \quad \text{Eq 10}$$

where, $(N_A)_0$ is the initial A-atom concentration at $t = 0$.

$$\begin{aligned}
N_B = N_B^0 &+ \frac{k_0 \gamma \lambda_n \lambda_A}{\lambda_n^2 + \omega^2} \left[\frac{\lambda_n \lambda_A - \omega^2}{(\lambda_A^2 + \omega^2)(\lambda_B^2 + \omega^2)} (\lambda_B \sin \omega t - \omega \cos \omega t) \right. \\
&- \frac{\omega(\lambda_n + \lambda_A)}{(\lambda_A^2 + \omega^2)(\lambda_B^2 + \omega^2)} (\lambda_B \cos \omega t + \omega \sin \omega t) + \frac{\omega}{(\lambda_A - \lambda_n)(\lambda_B - \lambda_n)} e^{-\lambda_n t} \Big] \\
&+ Y e^{-\lambda_A t} + (M+N) e^{-\lambda_B t}
\end{aligned} \tag{Eq 11}$$

$$\text{where, } Y = \frac{k_0 \gamma \lambda_n \lambda_A \omega}{(\lambda_n^2 + \omega^2)(\lambda_B - \lambda_A)} \left[\frac{\lambda_n + \lambda_A}{\lambda_A^2 + \omega^2} - \frac{1}{\lambda_A - \lambda_n} \right] \tag{Eq 12}$$

$$M = -Y \tag{Eq 13}$$

$$N = \frac{k_0 \gamma \lambda_n \lambda_A \omega}{\lambda_n^2 + \omega^2} \left[\frac{(\lambda_n \lambda_A - \omega^2) + (\lambda_n + \lambda_A) \lambda_B}{(\lambda_A^2 + \omega^2)(\lambda_B^2 + \omega^2)} - \frac{1}{(\lambda_A - \lambda_n)(\lambda_B - \lambda_n)} \right] \tag{Eq 14}$$

and where N_B^0 represents the solution for N_B for the unperturbed case given by (7):

$$\begin{aligned}
N_B^0 = \frac{k_0 \lambda_A \lambda_n}{\lambda_n} &\left[\frac{1}{\lambda_A \lambda_B} (1 - e^{-\lambda_B t}) + \frac{\lambda_n}{\lambda_A (\lambda_A - \lambda_n) (\lambda_B - \lambda_A)} (e^{-\lambda_A t} - e^{-\lambda_B t}) \right. \\
&- \left. \frac{1}{(\lambda_A - \lambda_n) (\lambda_B - \lambda_n)} (e^{-\lambda_n t} - e^{-\lambda_B t}) \right] + B
\end{aligned}$$

$$\begin{aligned}
\text{where, } B = \frac{(N_B)_0 \lambda_A \lambda_n}{\lambda_A - \lambda_n} &\left[\frac{1}{\lambda_B - \lambda_n} (e^{-\lambda_n t} - e^{-\lambda_B t}) + \frac{1}{\lambda_B - \lambda_A} (e^{-\lambda_B t} - e^{-\lambda_A t}) \right] \\
&+ \frac{(N_A)_0 \lambda_A}{\lambda_B - \lambda_A} (e^{-\lambda_A t} - e^{-\lambda_B t}) + (N_B)_0 e^{-\lambda_B t}
\end{aligned} \tag{Eq 15}$$

where $(N_B)_0$ is the initial B-atom concentration at $t = 0$.

$$\begin{aligned}
N_C = N_C^0 &+ \frac{k_0 \gamma \lambda_n \lambda_A \lambda_B}{\lambda_n^2 + \omega^2} \left[b(\lambda_C \sin \omega t - \omega \cos \omega t) \right. \\
&- c(\lambda_C \cos \omega t + \omega \sin \omega t) \\
&+ \left. d e^{-\lambda_n t} \right] \\
&+ \left(\frac{\lambda_B}{\lambda_C - \lambda_A} \right) Y e^{-\lambda_A t} + \left[X + M \left(\frac{\lambda_B}{\lambda_C - \lambda_B} \right) \right] e^{-\lambda_B t} \\
&+ \left[\frac{k_0 \gamma \lambda_n \lambda_A \lambda_B}{\lambda_n^2 + \omega^2} (b\omega + c + d) - \left(X + \frac{\lambda_B Y}{\lambda_C - \lambda_A} + \frac{\lambda_B M}{\lambda_C - \lambda_B} \right) \right] e^{-\lambda_C t}
\end{aligned} \tag{Eq 16}$$

where N_C^0 represents the solution to the unperturbed case for N_C given by (7):

$$\begin{aligned}
N_C^0 = & \frac{k_0 \lambda_A \lambda_B \lambda_n}{\lambda_n} \left[\frac{1}{\lambda_A \lambda_B \lambda_C} (1 - e^{-\lambda_C t}) - \frac{1}{\lambda_A \lambda_B (\lambda_C - \lambda_B)} (e^{-\lambda_B t} - e^{-\lambda_C t}) \right. \\
& + \frac{\lambda_n}{\lambda_A (\lambda_A - \lambda_n) (\lambda_B - \lambda_A) (\lambda_C - \lambda_B)} (e^{-\lambda_A t} - e^{-\lambda_C t}) \\
& - \frac{\lambda_n}{\lambda_A (\lambda_A - \lambda_n) (\lambda_B - \lambda_A) (\lambda_C - \lambda_B)} (e^{-\lambda_B t} - e^{-\lambda_C t}) \\
& - \frac{1}{(\lambda_A - \lambda_n) (\lambda_B - \lambda_n) (\lambda_C - \lambda_n)} (e^{-\lambda_n t} - e^{-\lambda_C t}) \\
& \left. + \frac{1}{(\lambda_A - \lambda_n) (\lambda_B - \lambda_n) (\lambda_C - \lambda_B)} (e^{-\lambda_B t} - e^{-\lambda_C t}) \right] + C
\end{aligned}$$

$$\begin{aligned}
\text{where, } C = & \lambda_n \lambda_A \lambda_B (N_n)_0 \left[\frac{e^{-\lambda_n t} - e^{-\lambda_C t}}{(\lambda_A - \lambda_n) (\lambda_B - \lambda_n) (\lambda_C - \lambda_n)} + \frac{e^{-\lambda_A t} - e^{-\lambda_C t}}{(\lambda_n - \lambda_A) (\lambda_B - \lambda_A) (\lambda_C - \lambda_A)} \right. \\
& + \left. \frac{e^{-\lambda_B t} - e^{-\lambda_C t}}{(\lambda_B - \lambda_n) (\lambda_B - \lambda_A) (\lambda_C - \lambda_B)} \right] + \frac{\lambda_A \lambda_B (N_A)_0}{(\lambda_B - \lambda_A) (\lambda_C - \lambda_A)} (e^{-\lambda_A t} - e^{-\lambda_C t}) \\
& + \frac{\lambda_A \lambda_B (N_A)_0}{(\lambda_B - \lambda_A) (\lambda_C - \lambda_B)} (e^{-\lambda_C t} - e^{-\lambda_B t}) + \frac{\lambda_B (N_B)_0}{\lambda_C - \lambda_B} (e^{-\lambda_B t} - e^{-\lambda_C t}) \\
& + (N_C)_0 e^{-\lambda_C t}
\end{aligned}$$

Eq 17

where $(N_C)_0$ is the initial C-atom concentration at $t = 0$.

The other constants in Equation 16 are as follows:

$$b = \frac{(\lambda_n \lambda_A - \omega^2) \lambda_B - \omega^2 (\lambda_n + \lambda_A)}{(\lambda_A^2 + \omega^2) (\lambda_B^2 + \omega^2) (\lambda_C^2 + \omega^2)} \quad \text{Eq 18}$$

$$c = \frac{\omega [(\lambda_n \lambda_A - \omega^2) + (\lambda_n + \lambda_A) \lambda_B]}{(\lambda_A^2 + \omega^2) (\lambda_B^2 + \omega^2) (\lambda_C^2 + \omega^2)} \quad \text{Eq 19}$$

$$d = \frac{\omega}{(\lambda_A - \lambda_n) (\lambda_B - \lambda_n) (\lambda_C - \lambda_n)} \quad \text{Eq 20}$$

$$x = \frac{k_0 \gamma \lambda_n \lambda_A \lambda_B \omega}{(\lambda_n^2 + \omega^2) (\lambda_C - \lambda_B)} \left[\frac{(\lambda_n \lambda_A - \omega^2) + (\lambda_n + \lambda_A) \lambda_B}{(\lambda_A^2 + \omega^2) (\lambda_B^2 + \omega^2)} - \frac{1}{(\lambda_A - \lambda_n) (\lambda_B - \lambda_n)} \right] \quad \text{Eq 21}$$

and finally,

$$N_C' = N_C (\lambda_C / \lambda_C') \text{ for RaC}', \text{ and} \quad \text{Eq 22}$$

$$N_C' = 0.663 N_C (\lambda_C / \lambda_C') \text{ for ThC}' \quad \text{Eq 23}$$

LINEAR PERTURBATION

The solutions for $k_n = k_0(1+nt)$ are as follows:

$$N_n = N_n^0 + \frac{k_0 n}{\Lambda_n^2} \left[(\Lambda_n t - 1) + e^{-\Lambda_n t} \right] \quad \text{Eq 24}$$

where N_n^0 is given by Equation 6.

$$N_A = N_A^0 + \frac{k_0 n \lambda_n}{\Lambda_n^2} \left[\frac{\Lambda_n}{\Lambda_A^2} (\Lambda_A t - 1) - \frac{1}{\Lambda_A} + \frac{e^{-\Lambda_n t}}{\Lambda_A - \Lambda_n} \right] + \frac{k_0 n \lambda_n}{\Lambda_A^2 (\Lambda_n - \Lambda_A)} e^{-\Lambda_A t} \quad \text{Eq 25}$$

where N_A^0 is given by Equation 9.

$$N_B = N_B^0 + \frac{k_0 n \lambda_n \lambda_A}{\Lambda_n^2} \left[\frac{\Lambda_n}{\Lambda_A \Lambda_B^2} (\Lambda_B t - 1) - \frac{1}{\Lambda_A \Lambda_B} \left(\frac{\Lambda_n}{\Lambda_A} + 1 \right) + \frac{e^{-\Lambda_n t}}{(\Lambda_A - \Lambda_n)(\Lambda_B - \Lambda_n)} \right] + \frac{k_0 n \lambda_n \lambda_A}{\Lambda_A^2 (\Lambda_n - \Lambda_A)(\Lambda_B - \Lambda_A)} e^{-\Lambda_A t} + \frac{k_0 n \lambda_n \lambda_A}{\Lambda_n^2} f e^{-\Lambda_B t} \quad \text{Eq 26}$$

where N_B^0 is given by Equation 15, and

$$f = \left[\frac{1}{\Lambda_A \Lambda_B} \left(\frac{\Lambda_n}{\Lambda_B} + \frac{\Lambda_n}{\Lambda_A} + 1 \right) - \frac{1}{(\Lambda_A - \Lambda_n)(\Lambda_B - \Lambda_n)} - \frac{\Lambda_n^2}{\Lambda_A^2 (\Lambda_n - \Lambda_A)(\Lambda_B - \Lambda_A)} \right] \quad \text{Eq 27}$$

$$N_C = N_C^0 + \frac{k_0 n \lambda_n \lambda_A \lambda_B}{\Lambda_n^2} \left[\frac{\Lambda_n}{\Lambda_A \Lambda_B \Lambda_C} (\Lambda_C t - 1) - \frac{1}{\Lambda_A \Lambda_B \Lambda_C} \left(\frac{\Lambda_n}{\Lambda_B} + \frac{\Lambda_n}{\Lambda_A} + 1 \right) + \frac{e^{-\Lambda_n t}}{(\Lambda_A - \Lambda_n)(\Lambda_B - \Lambda_n)(\Lambda_C - \Lambda_n)} \right] + \frac{k_0 n \lambda_n \lambda_A \lambda_B}{\Lambda_A^2 (\Lambda_n - \Lambda_A)(\Lambda_B - \Lambda_A)(\Lambda_C - \Lambda_A)} e^{-\Lambda_A t} + \frac{k_0 n \lambda_n \lambda_A \lambda_B}{\Lambda_n^2 (\Lambda_C - \Lambda_B)} f e^{-\Lambda_B t} + \left(g - \frac{f}{\Lambda_C - \Lambda_B} \right) e^{-\Lambda_C t} \quad \text{Eq 28}$$

where N_C^0 is given by Equation 17, and

$$g = \frac{k_0 n \lambda_n \lambda_A \lambda_B}{\Lambda_n^2} \left[\frac{1}{\Lambda_A \Lambda_B \Lambda_C} \left(\frac{\Lambda_n}{\Lambda_A} + \frac{\Lambda_n}{\Lambda_B} + \frac{\Lambda_n}{\Lambda_C} + 1 \right) - \frac{1}{(\Lambda_A - \Lambda_n)(\Lambda_B - \Lambda_n)(\Lambda_C - \Lambda_n)} \right] - \frac{k_0 n \lambda_n \lambda_A \lambda_B}{\Lambda_A^2 (\Lambda_n - \Lambda_A)(\Lambda_B - \Lambda_A)(\Lambda_C - \Lambda_A)} \quad \text{Eq 29}$$

The expressions for N_C' are the same as those given by Equations 22 and 23.

EXPONENTIAL PERTURBATION

The solutions for $k_n = k_0 e^{\delta t}$ are as follows:

$$N_n = \frac{k_0}{\delta + \Lambda_n} (e^{\delta t} - e^{-\Lambda_n t}) + (N_n)_0 e^{-\Lambda_n t} \quad \text{Eq 30}$$

$$N_A = \ell e^{\delta t} + m e^{-\Lambda_n t} + [(N_A)_0 - h] e^{-\Lambda_A t} \quad \text{Eq 31}$$

where,

$$h = \frac{k_0 \lambda_n}{\delta + \Lambda_n} \left(\frac{1}{\delta + \Lambda_A} - \frac{1}{\Lambda_A - \Lambda_n} \right) + \frac{(N_n)_0 \lambda_n}{\Lambda_A - \Lambda_n}, \quad \text{Eq 32}$$

$$\ell = \frac{k_0 \lambda_n}{(\delta + \Lambda_n)(\delta + \Lambda_A)}, \text{ and}$$

$$m = \frac{(N_n)_0 \lambda_n}{\Lambda_A - \Lambda_n} - \frac{k_0 \lambda_n}{(\delta + \Lambda_n)(\Lambda_A - \Lambda_n)} \quad \text{Eq 33}$$

and, where $(N_n)_0$ and $(N_A)_0$ in Equations 30 and 31 have their usual meaning as described above.

$$N_B = \frac{\ell \lambda_A}{\delta + \Lambda_B} e^{\delta t} + \frac{m \lambda_A}{\Lambda_B - \Lambda_n} e^{-\Lambda_n t} + \left[\frac{(N_A)_0 - h}{\Lambda_B - \Lambda_A} \right] e^{-\Lambda_A t} + [(N_B)_0 - p] e^{-\Lambda_B t} \quad \text{Eq 34}$$

where,

$$p = \frac{\ell \lambda_A}{\delta + \Lambda_B} + \frac{m \lambda_A}{\Lambda_B - \Lambda_n} + \left[\frac{(N_A)_0 - h}{\Lambda_B - \Lambda_A} \right] \quad \text{Eq 35}$$

where $(N_B)_0$ in Equation 34 has its usual meaning as indicated above.

$$N_C = q e^{\delta t} + r e^{-\Lambda_n t} + s e^{-\Lambda_A t} + v e^{-\Lambda_B t} + [(N_C)_0 - u] e^{-\Lambda_C t} \quad \text{Eq 36}$$

where,

$$q = \frac{\ell \lambda_A \lambda_B}{(\delta + \Lambda_B)(\delta + \Lambda_C)}, \quad \text{Eq 37}$$

$$r = \frac{m \lambda_A \lambda_B}{(\Lambda_B - \Lambda_n)(\Lambda_C - \Lambda_n)}, \quad \text{Eq 38}$$

$$s = \frac{[(N_A)_0 - h] \lambda_B}{(\Lambda_B - \Lambda_A)(\Lambda_C - \Lambda_A)}, \text{ and} \quad \text{Eq 39}$$

$$u = q + r + s + v \quad \text{Eq 40}$$

where $(N_C)_0$ has its usual meaning as indicated for the case of other perturbations.

The expressions for N_C' are those corresponding to Equations 22 and 23.

DISCUSSION

The expressions for N_i and N_n for the different perturbation cases investigated here are quite complex algebraically. It is not difficult to show that the solutions to Equations 1 and 2 exhibit certain 'symmetries' which are not always apparent or reflected in the particular form given here, where in some instances, symmetry and/or 'beauty' has been sacrificed in favour of practicality.

As expected, the solutions for N_n and N_i are proportional to k_0 , i.e., the unperturbed emanation coefficient. Furthermore, the following observations for N_n and N_i for all perturbation cases are worthwhile noticing. For any given perturbation case, the solutions to Equations 1 and 2:

- a) contain terms proportional to the perturbation and its time derivative;
- b) contain terms corresponding to the initial condition, i.e., for $t = 0$ multiplied by a decay factor of the form $e^{-\lambda_m t}$, where $m = n$ or i ;
- c) contain exponential decay terms ranging from the radioactive gas to the particular radionuclide of interest.

In general terms, and from items a), b) and c), it can be surmised that the solutions to Equations 1 and 2 exhibit a similar, although somewhat modified, behaviour to that corresponding to the perturbation. Furthermore, because of the time-dependent nature of the perturbation, radioactive decay, and the different half-lives of the radionuclides involved, N_n and N_i are out of phase as shown elsewhere (7,8).

Data pertaining to radon and thoron activity, radon and thoron progeny activity and Working Level, and some radon and thoron progeny ratios of considerable practical interest have been presented elsewhere (7,8).

CONCLUDING REMARKS

Because conditions in underground uranium mines rarely remain constant, it is important to investigate radiation levels under variable conditions. However, it is in most cases difficult to predict the extent and kind of variations to which some variables, that determine radiation levels, are subjected. For these, and other reasons, it is of practical, as well as theoretical, interest to investigate the behaviour of mine models under varying, but predictable, well defined, conditions as those discussed here. It is found in practice that most underground variations can be approximated by one, or a combination of several, of the cases presented here. It is also found that radiation levels are very much affected by the type and extent of the 'perturbation' introduced in the emanation rate or ventilation rate. These changes are important for environmental monitoring and personal dosimetry purposes as they may lead to erroneous conclusions regarding environmental levels and personal dose exposure.

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